# NUMBER PATTERNS AND SEQUENCES: BASICS OF MATHEMATICAL PATTERNS

# CHAPTER 2: ARITHMETIC AND GEOMETRIC SEQUENCES

## 2.1. Introduction

In the domain of mathematical patterns and sequences, two fundamental structures stand as bases of numerical organization: the Arithmetic Sequence and the Geometric Sequence. These precise constructs, deeply rooted in mathematics, serve as the building blocks for numerous mathematical and real-world phenomena (Rahmani-Andebili, 2021).

Arithmetic sequences provide a foundational understanding of how numbers evolve in a predictable and consistent manner. By encapsulating the concept of a common difference between consecutive terms, arithmetic sequences teach us to recognize and work with the fundamental idea of linear growth. This understanding extends beyond mathematics, influencing comprehension of rates of change and gradual progression in various aspects of the physical world. Furthermore, arithmetic sequences serve as a gateway to more advanced mathematical concepts, such as calculus, where they underpin the understanding of limits and derivatives. In the practical realm, they find applications in fields like finance and engineering, aiding in the modeling of linearly changing quantities. As an educational tool, arithmetic sequences help learners develop problem-solving skills and mathematical reasoning, making them an essential component of mathematical education (Rahmani-Andebili, 2021).

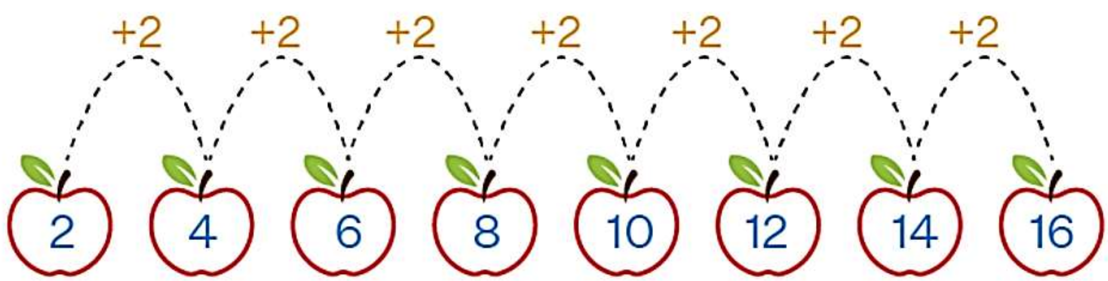



***Figure 2.1. Illustration of an arithmetic sequence (Source: Cuemath, Creative Commons License)***

Geometric sequences offer a unique perspective on the progression of numbers, demonstrating the power of exponential growth or decay. The common ratio between consecutive terms in a geometric sequence highlights the concept of multiplication as the driving force behind this growth. In mathematics, geometric sequences provide insights into exponential functions and the concept of limits, which are crucial in calculus. Moreover, they play a vital role in finance, modeling scenarios like compound interest and exponential growth in investments. In physics and engineering, geometric sequences help describe processes with exponential behavior, such as radioactive decay. In computer science, they are essential in algorithms, particularly those involving exponential time complexity.

Beyond these applications, geometric sequences foster a deep understanding of exponential phenomena and serve as a fundamental educational tool, helping students grasp exponential functions, geometric series, and the significance of exponential growth in various real-world contexts (Camouzis & Kotsios, 2021).

## 2.2. Arithmetic Sequences

In mathematical sequences, the arithmetic sequence holds a central position as a structured and fundamental pattern. As already discussed in chapter 1, an arithmetic sequence, often referred to as an arithmetic progression, is a sequence of numbers in which the difference between consecutive terms remains constant. This constant difference is precisely defined as the "common difference" and is denoted by the symbol 'd'. Arithmetic sequences play a pivotal role in various mathematical contexts and find applications in fields such as finance, physics, and computer science, where linear and predictable changes are encountered (Chapman, Gotti, & Gotti, 2020).

Mathematically, an arithmetic sequence can be precisely defined through the following formula:

$$a_n = a_1 + (n-1)d$$

Here, the variables are as follows:

i. $a_n$ represents the $n$th term of the sequence.

ii. $a_1$ denotes the first term of the sequence.

iii. $n$ stands for the position of the term in the sequence.

iv. $d$ signifies the common difference between consecutive terms.

The essence of an arithmetic sequence lies in its predictability. As one progresses from term to term, the common difference 'd' is consistently added to the previous term, ensuring uniformity throughout the sequence (Siagian, Aswin, & Herman, 2023).

Illustrating this with a mathematical example:

**Example: Calculating the 20$^{th}$ Term**

Suppose an arithmetic sequence is given with the first term $a_1$ equal to 3 and a common difference $d$ of 4. To compute the 20$^{th}$ term, the arithmetic sequence formula is employed.

Using the formula:

$$a_{20} = a_1 + (20-1) \cdot d$$

Substituting the given values:

$$a_{20} = 3 + (20 - 1) \cdot 4$$

Simplifying the expression within the parentheses:

$$a_{20} = 3 + 19 \cdot 4$$

Calculating the product:

$$a_{20} = 3 + 76$$

Finally, summing the values:

$$a_{20} = 79$$

Hence, the 20th term of this arithmetic sequence is determined to be 79.

Finding any specific term in an arithmetic sequence, such as the 20th term, is crucial for a variety of reasons. Firstly, it serves as a validation point, confirming that the sequence maintains its consistent pattern over the desired number of terms. This verification of pattern is not limited to the 20th term but can be applied to any term of interest. Additionally, by choosing a specific term, you gain insights into the sequence's behavior at that point in its progression. This understanding can be invaluable in real-world applications, such as financial planning or modeling, where you need to project future values based on the established pattern. Furthermore, selecting any term in the sequence requires the comprehensibility of mathematical concepts, making them more accessible and illustrative, especially in educational contexts (Rahmani-Andebili, 2021).

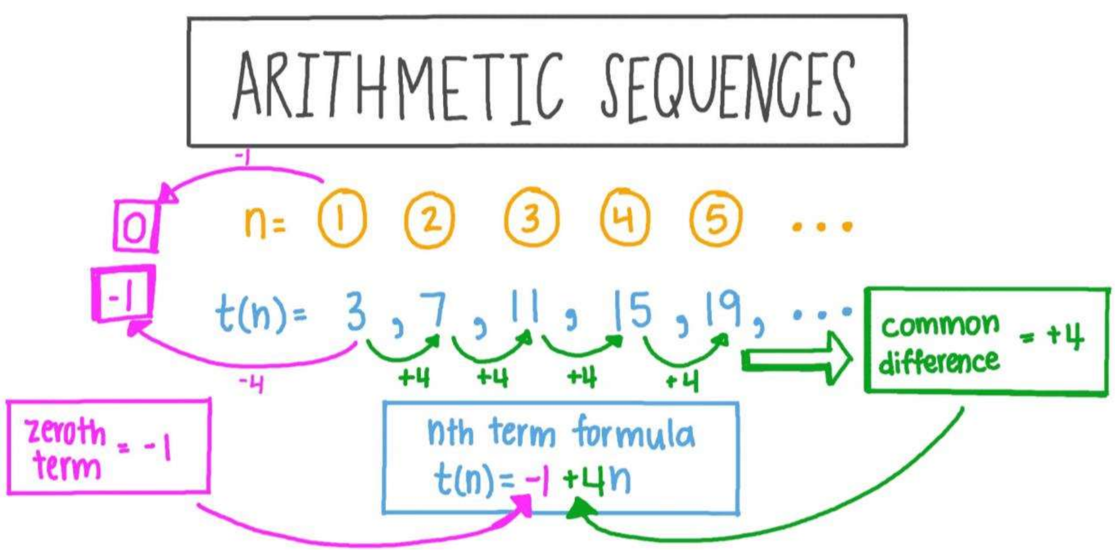



***Figure 2.2. Example of an arithmetic sequence (Source: Nagwa, Creative Commons License)***

### 2.2.1. The Common Difference

The concept of the 'common difference' is significant in the context of arithmetic sequences. In the following sections, we will discuss this mathematical concept, thoroughly defining it, examining its implications, and discussing its intricacies through illustrative examples (Alexan et al., 2018).

**Definition and Mathematical Expression:** In the arithmetic sequence: $a_n = a_1 + (n-1)d$, each term differs from the previous one by a fixed constant. This constant is what we refer to as the ‘common difference’ denoted by $(d)$ (Ross, 2017).

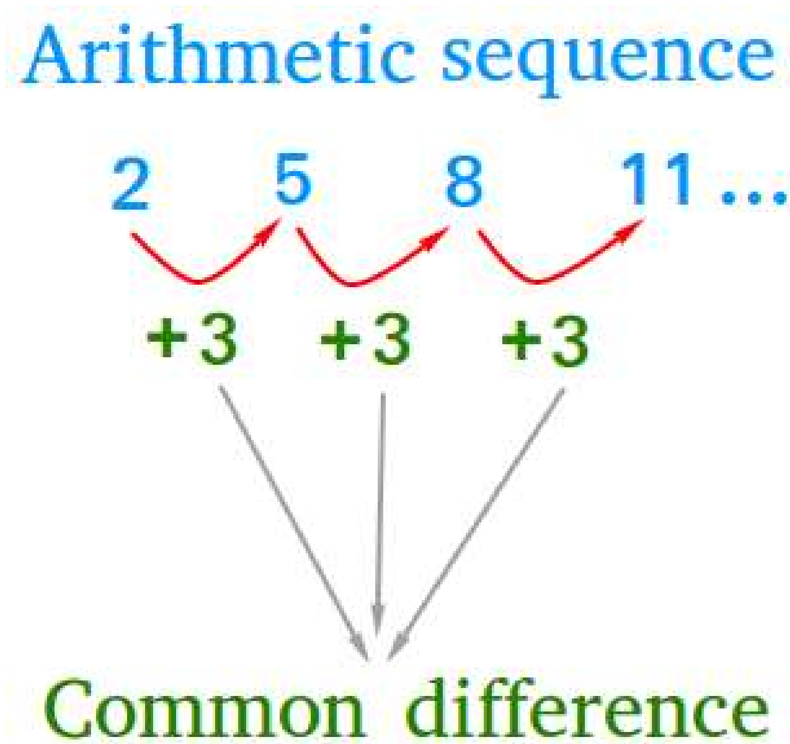



*Figure 2.3. Schematic of common difference of arithmetic mean (Source: Math-dictionary, Creative Commons License)*

#### 2.2.1.1. Visualizing the Linear Progression

Suppose a sequence of numbers where each term increases or decreases by the same constant number. This gradual shift creates a visually perceptible line when plotted on a graph. As each term is connected to the next through the common difference, the sequence unfolds linearly.

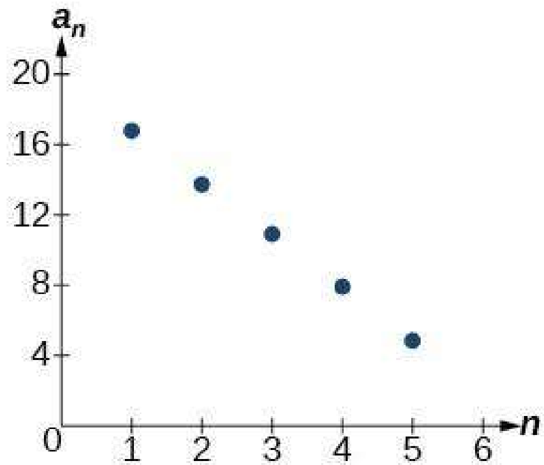



***Figure 2.4. Graph illustrating tan arithmetic sequence with the common difference of -3 (Source: Lumen Learning, Creative Commons License)***

The above graph shows an arithmetic sequence with $a_1 = \ 17 \ and \ d = -3$.'

**Calculating Terms Using the Common Difference:**' Consider the arithmetic sequence $(5, 9, 13, 17, \dots)$ with a common difference 4. We wish to determine the 10th term of this sequence using the common difference formula.

Using the formula of an arithmetic sequence,

$$a_{10} = 5 + (10 - 1) \cdot 4 = 5 + 36 = 41$$

Hence, the 10th term of this sequence is 41, showcasing the consistent progression governed by the common difference.

### 2.2.1.2. Applications in Real-world Scenarios

The common difference finds its application in various real-world contexts. Consider financial investments where the value increases or decreases by a fixed amount periodically. This incremental change aligns with the arithmetic sequence's linear growth pattern, wherein the constant difference represents the regular increment or decrement.

For instance, if an individual invests $100 in a savings account with an annual interest rate of 5%, the balance at the end of each year forms an arithmetic sequence. The common difference in this example is $5, representing the annual interest earned.

The common difference concept enables us to address various problems involving arithmetic sequences. Whether it is predicting future terms, determining past terms, or understanding the relationship between different terms, the common difference acts as a guiding principle.

### 2.2.2. Finding the n^th^ Term of an Arithmetic Sequence

Within the domain of arithmetic sequences, the determination of the $n^{th}$ term presents itself as a mathematical pursuit like the solving of a cryptic problem. This $n^{th}$ term holds the key to comprehending the ordered advancement of numbers within the sequence.

#### 2.2.2.1. The Importance of the $n^{th}$ Term

The $n^{th}$ term of an arithmetic sequence serves as a pivotal tool for various mathematical and real-world applications. It allows us to predict or determine the value of any term in the sequence without the need to calculate all the preceding terms individually. This efficiency is especially valuable when dealing with lengthy sequences.

**Illustrative Example:** Consider the following arithmetic sequence: 7, 11, 15, 19, 23, ... It is clear that the common difference, 'd', between each term is 4. We want to find the $10^{th}$ term of this sequence $(a_{10})$.

Using the nth term formula:

$$a_n = a_1 + (n - 1)d$$

We can plug in the values for this sequence:

$a_1 = 7$ (the first term)

$n = 10$ (we want to find the 10th term)

$d = 4$ (the common difference)

Now, let's calculate $(a_{10})$:

$$a_{10} = 7 + (10 - 1) \cdot 4$$

$$a_{10} = 7 + 9 \cdot 4$$

$$a_{10} = 7 + 36$$

$$a_{10} = 43$$

So, the $10^{th}$ term $(a_{10})$ of this arithmetic sequence is 43.

#### 2.2.2.2. Insight into the Formula

To gain a deeper understanding of why this formula works, let's break it down step by step.

$a_1$: This is the first term of the sequence and serves as the starting point.

$n - 1$: This part accounts for how many "steps" we need to take within the sequence to reach the desired term. Since we're finding the nth term, we need to take $(n - 1)$ steps because we start from the first term.

$d$: The common difference, $(d)$, represents the magnitude of each step. Multiplying $(n-1)$by $(d)$ ensures we take the correct number of steps in the correct direction.

$a_n$: This is the nth term we are seeking, and it is obtained by adding the number of steps$(n-1)d$ to the starting point $(a_1)$.

In essence, the formula accounts for the cumulative effect of taking a certain number of steps $(n - 1)$ of a consistent size $(d)$ from the starting point $(a_1)$ to reach the nth term $(a_n)$.

#### 2.2.2.3. Applications in Real-world Scenarios

The ability to find the $n^{th}$ term of an arithmetic sequence is not confined to mathematical exercises alone. It finds extensive applications in various real-world scenarios. Consider financial calculations, where periodic increments or decrements occur. For instance, if you have a savings account with an initial balance $(a_1)$ and a fixed annual interest rate $(d)$, you can use the $n^{th}$ term formula to predict the balance $(a_n)$ after a certain number of years $(n)$. This is a practical application of arithmetic sequences in finance (Firdaus et al., 2020).

Without the $n^{th}$ term formula, calculating a specific term in a long arithmetic sequence could be a laborious task, especially for sequences with thousands or even millions of terms. With this formula, you can swiftly determine any term of interest.

The formula for finding the nth term of an arithmetic sequence is a powerful tool in mathematics and practical applications. It simplifies the process of predicting or determining specific terms within a sequence characterized by a constant common difference (Sarker, 2021).

## 2.3. Arithmetic Mean and Series

In mathematics, arithmetic means and arithmetic series are fundamental concepts that help us understand how numbers balance and add up in a sequence. This exploration aims to break down the arithmetic mean, dig into the arithmetic series, and show their importance through math formulas and real-world examples.

### 2.3.1. The Arithmetic Mean

The arithmetic mean, often simply referred to as the "average," is a fundamental concept in mathematics. It provides a central value that represents a set of numbers in such a way that the sum of deviations from this value is zero. In other words, it is the balance point of a data set. The arithmetic mean of a set of numbers is calculated by adding up all the numbers and then dividing by the count of numbers in the set (Casey et al., 2004).

Mathematically, the arithmetic mean $(\bar{x})$ of a set of $(n)$ numbers $(x_1, x_2, x_3, \dots, x_n)$is expressed as:

$$\bar{x} = \frac{1}{n}(x_1 + x_2 + x_3 + \cdots + x_n)$$

This formula finds its practical utility in various fields, such as statistics, finance, and science, where understanding the central tendency of data is paramount.

### 2.3.2. The Arithmetic Series

An arithmetic series, on the other hand, is the summation of the terms of an arithmetic sequence. It provides a concise representation of the cumulative effect of an ordered list of numbers that progress uniformly with a common difference (Alghar et al., 2022; Anjum & Mukherjee, 2022).

The sum of the first $(n)$ terms of an arithmetic series, denoted as $(S_n)$, can be expressed as follows:

$$S_n = n \cdot \bar{x}$$

Where:

$S_n$ represents the sum of the first $(n)$ terms.

$n$ is the number of terms.

$\bar{x}$ is the arithmetic mean of the series.

This formula offers a straightforward means of calculating the total value of an arithmetic series, making it a valuable tool in various mathematical and real-world contexts.

**Example: Calculating the Arithmetic Mean**

Consider a set of exam scores: 85, 90, 78, 92, and 88. We want to find the arithmetic mean of these scores.

Using the arithmetic mean formula:

$$\bar{x} = \frac{1}{n}(x_1 + x_2 + x_3 + x_4 + x_5)$$

Where $(n)$ is the count of numbers in the set (in this case $n = 5$, and $x_1, x_2, x_3, x_4, x_5$ are the individual scores, we can calculate the mean as follows:

$$\bar{x} = \frac{1}{5}(85 + 90 + 78 + 92 + 88)$$

Now, add up the numbers:

$$\bar{x} = \frac{1}{5}(433)$$

Finally, calculate the mean:

$$\bar{x} = \frac{433}{5} = 86.6$$

So, the arithmetic mean of these exam scores is 86.6.

**Example: Calculating the Arithmetic Series**

Suppose the car's speedometer readings at regular intervals are as follows: 60 km/h, 65 km/h, 70 km/h, and 75 km/h. We want to find the total distance covered in 4 hours.

First, let's calculate the arithmetic mean of the speeds using the formula we discussed earlier. In this case, $(n = 4)$, and the speeds are 60, 65, 70, and 75 km/h:

$$\bar{x} = \frac{1}{4}(60 + 65 + 70 + 75) = \frac{1}{4}(270) = 67.5 \text{ km/h}$$

Now that we have the arithmetic mean of 67.5 km/h, we can calculate the total distance $(S_n)$ covered in 4 hours using the arithmetic series formula:

$$S_n = n \cdot \bar{x} = 4 \cdot 67.5 = 270 \text{ km}$$

So, the car covered a total distance of 270 kilometers in 4 hours.

### 2.3.3. Applications of Arithmetic Mean and Series

The concepts of arithmetic mean and arithmetic series find application in various fields (Pickover, 2011):

1. **Statistics:** In data analysis, the arithmetic mean is a fundamental measure of central tendency, providing insights into the average value of a dataset.
2. **Finance:** In finance, the arithmetic means is used to calculate returns on investments, helping investors understand the average return on their portfolio.
3. **Physics:** In physics, the arithmetic mean is employed to calculate average velocities and accelerations (Joan, 2015).
4. **Engineering:** In engineering, the arithmetic mean is used to determine the average stress or strain in materials.

5. **Economics:** In economics, the arithmetic mean is used to calculate indices like the Consumer Price Index (CPI), reflecting the average change in prices over time.

6. **Distance and Speed Calculations:** As demonstrated in the car trip example, the arithmetic mean and series help calculate total distances and cumulative effects of varying speeds (Grassi, 2021; Sun & Chen, 2018).

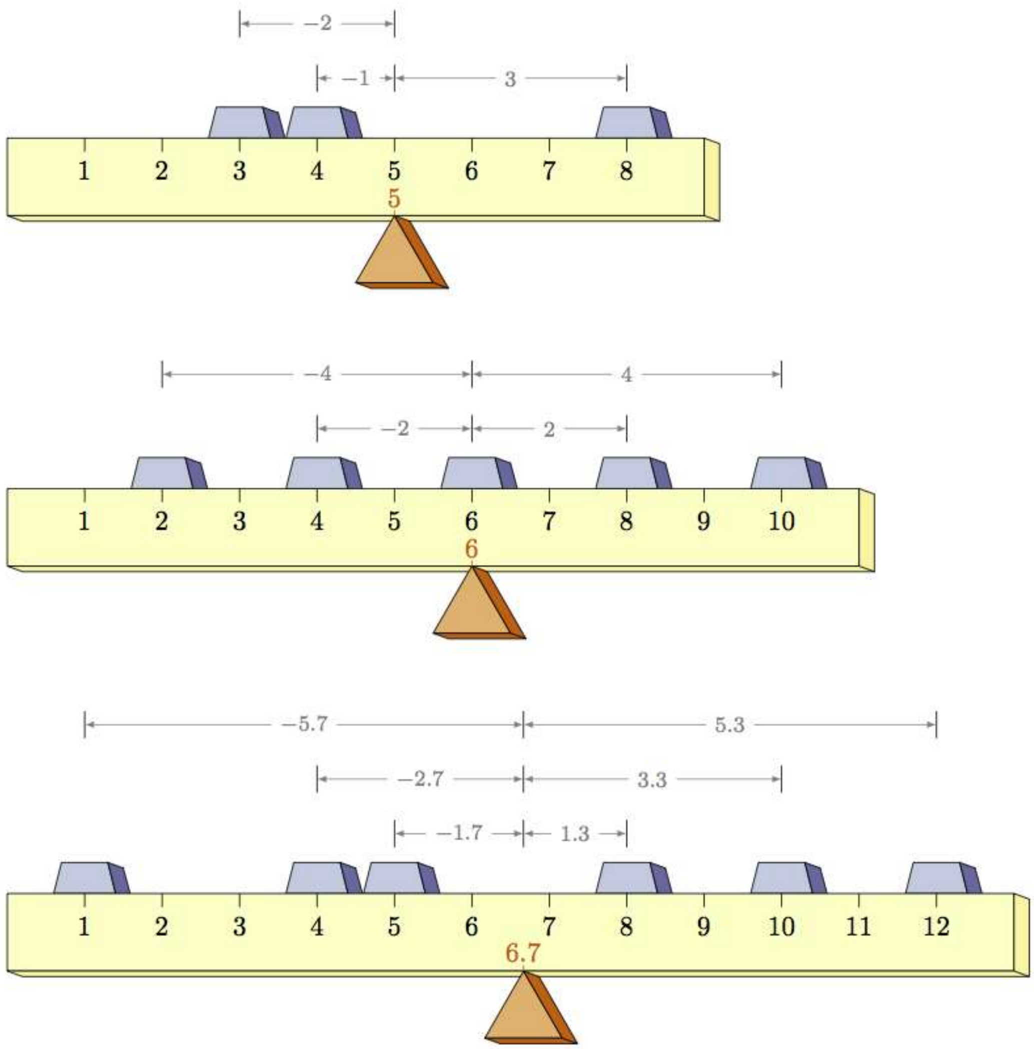



***Figure 2.5. The arithmetic mean can be visualized as a balancing point on a scale (Source: Brillinat.org, Creative Commons License)***

### 2.3.4. Arithmetic Mean and Series in Finance

Arithmetic sequences find invaluable applications in finance, where they help model and solve various financial scenarios. These sequences provide a structured approach to understanding how values change over time, particularly when there is a consistent change or growth in financial parameters (Wang et al., 2012).

**Remember:**

In finance, the arithmetic series plays a crucial role in calculating the total amount paid overtime for loans or investments with fixed, regular payments. It's a fundamental concept for financial planning and understanding the cost or return on investments.

**Arithmetic Mean:** It provides a central measure of a dataset and is used in finance to understand the typical or average value of financial data. For example, consider the monthly returns of a stock over a year: +2%, -1%, +4%, -3%, +5%, and so on. To find the average monthly return, you add up all these returns and divide them by the number of months (12, in this case). So, $(2 - 1 + 4 - 3 + 5 + \ldots)/12$ = Average Monthly Return.

**Arithmetic Series**: In finance, an arithmetic series is a summation of financial data that follows a consistent linear pattern. It is used to calculate the total amount or accumulated value of a series of cash flows over time, where each cash flow differs by a constant amount.

Suppose you are running a small business, and every month, you make a profit that increases by $500. In the first month, your profit is $1,000, and it increases by $500 each month. You want to calculate your total profit for the first six months using the arithmetic series formula.

Using the formula ($S_n = n \cdot \bar{x}$),

$n = 6$ (as we are calculating the first six months)

and

$$(\bar{x} = \frac{\text{First-month profit} + \text{Last month's profit}}{2} = \frac{1{,}000 + 3{,}000}{2} = 2{,}000)$$

Now, plug these values into the formula:

$$S_n = 6 \cdot 2{,}000 = 12{,}000$$

So, your total profit for the first six months would be $12,000.

Furthermore, arithmetic series in finance can be extended to calculate the total interest earned over a specific period. The total interest is the sum of the annual interest earned each year, which is a simple arithmetic series. For instance, if you want to know how much interest you will earn after 10 years, you can sum the first 10 terms of the arithmetic sequence, each representing the interest earned in a specific year (Gunnarsson et al., 2016).

Arithmetic sequences in finance offer a systematic way to model the growth or change of financial values over time. Whether it is calculating future investment values, projecting savings growth, or determining interest earnings, these sequences provide financial professionals and individuals with powerful tools for making informed financial decisions and achieving their financial goals (Aharonov & Stephenson, 1998).

## 2.4. Geometric Sequences

These sequences exhibit an inherent multiplicative pattern, showcasing exponential growth or decay. In this section, geometric sequences will be discussed, showing their fundamental characteristics, mathematical expressions, and practical applications.

A geometric sequence is an ordered list of numbers where each term is obtained by multiplying the previous term by a fixed constant known as the common ratio, often denoted as $(r)$. The common ratio dictates the progression of the sequence, making it either increase exponentially when $(r)$ is greater than 1 or decrease exponentially when $(r)$ is between 0 and 1 (Morton et al., 2017).

Mathematically, a geometric sequence can be defined as:

$$a_n = a_1 \cdot r^{(n-1)}$$

Here, $a_n$ represents the $n^{\text{th}}$ term of the sequence, $a_1$ is the first term, $r$ is the common ratio, and $n$ signifies the term number.

**Example: Geometric Growth of Bacteria**

To illustrate the concept of geometric sequences, consider a situation where a bacterium population doubles every hour. Starting with an initial population of 10 bacteria$(a_1 = 10)$, we can use a geometric sequence to model its growth (Gunnarsson et al., 2016).

The common ratio in this scenario is $(r = 2)$ because the population doubles each hour. Using the formula for a geometric sequence, we can calculate the population at different time intervals:

1. After 1 hour $(n = 2)$: $(a_2 = 10 \cdot 2^{(2-1)} = 20)$ bacteria.
2. After 2 hours$(n = 3)$: $(a_3 = 10 \cdot 2^{(3-1)} = 40)$ bacteria.
3. After 3 hours $(n = 4)$: $(a_4 = 10 \cdot 2^{(4-1)} = 80)$ bacteria.

This progression demonstrates the exponential growth characteristic of geometric sequences when the common ratio is greater than 1 (Amsari et al., 2022).

**Example: Geometric Decay**

Now, consider a scenario where the value of a car depreciates by 20% each year. Starting with a car's initial value of $20,000 ($a_1 = 20{,}000$), we can use a geometric sequence to model its depreciation.

The common ratio in this case is ($r = 0.8$) because the car's value decreases to 80% of its previous value each year. Using the formula for a geometric sequence, we can calculate the car's value after different years:

1. After 1 year ($n = 2$): ($a_2 = 20{,}000 \cdot 0.8^{(2-1)} = 16{,}000$) dollars.
2. After 2 years ($n = 3$): ($a_3 = 20{,}000 \cdot 0.8^{(3-1)} = 12{,}800$) dollars.
3. After 3 years ($n = 4$): ($a_4 = 20{,}000 \cdot 0.8^{(4-1)} = 10{,}240$) dollars.

This demonstrates the exponential decay behavior of geometric sequences when the common ratio is between 0 and 1.

**Applications in Real-world Scenarios:**

Geometric sequences find extensive application in various real-world contexts due to their ability to model exponential growth and decay. Here are a few examples:

1. **Finance:** In investments, geometric sequences help calculate compound interest, where the initial amount grows by a fixed percentage over time.

2. **Biology:** Geometric sequences are used to model population growth or decline in ecological studies, such as the spread of viruses or the decline of endangered species.

3. **Physics:** They play a role in modeling the exponential decay of radioactive substances, as well as in understanding processes like sound attenuation and light intensity.

4. **Engineering:** Engineers use geometric sequences to analyze signal processing, filter design, and the behavior of electrical circuits.

5. **Computer Science:** In algorithms and data structures, geometric sequences can represent the time complexity of algorithms with exponential growth or decay.

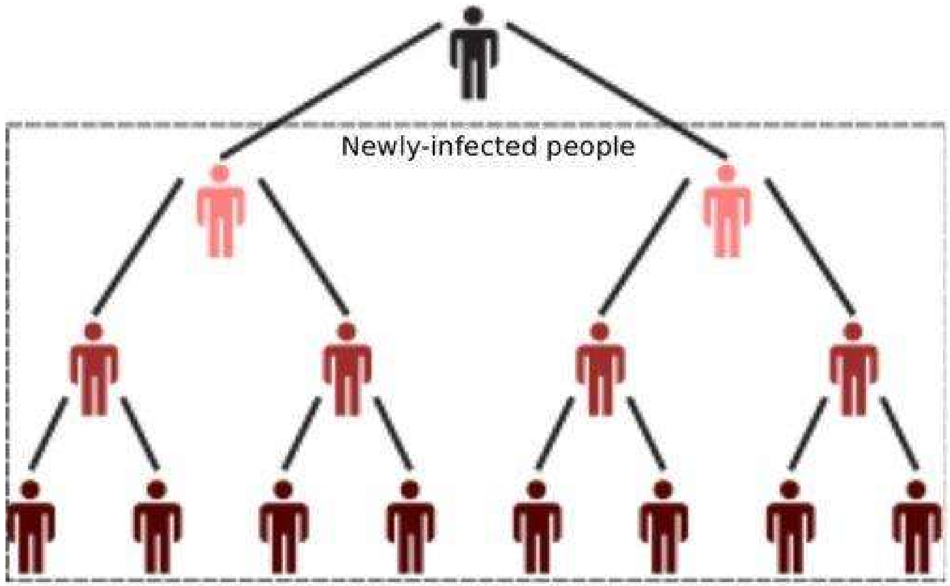



***Figure 2.6. An example of the geometric sequence where each person infects two more people with the flu virus(Source: Siyavula, Creative Commons License)***

**The Power of Geometric Sequences:**

Geometric sequences hold a unique place in the world of mathematical patterns. Their exponential nature allows them to accurately represent phenomena involving constant growth or decay. Understanding geometric sequences equips us with a powerful tool for predicting and modeling real-world scenarios, from investments to population dynamics, where the power of exponential change is at play (Shih & Lee, 2019).

## 2.4.1. The Common Ratio

Common ratio plays a pivotal role, guiding the sequence's behavior and dictating whether it showcases exponential growth or decay. In this section, we will discuss the concept of the common ratio and its mathematical implications and provide concrete examples to show its significance (Kondo & Miura, 2010).

### 2.4.1.1. Defining the Common Ratio

The common ratio $(r)$ in a geometric sequence is the fixed constant by which each term is obtained from the preceding one. In simpler terms, it represents the factor by which the sequence either grows or shrinks from term to term. This constant is integral to the geometric sequence's identity, as it fundamentally defines its progression.

Mathematically, for common ratio, we can use geometric sequence:

$$a_n = a_1 \cdot r^{(n-1)}$$

Here, ($a_n$) denotes the ($n$)-th term of the sequence, ($a_1$) signifies the first term, ($\boldsymbol{r}$) **is the common ratio**, and ($n$) represents the term number.

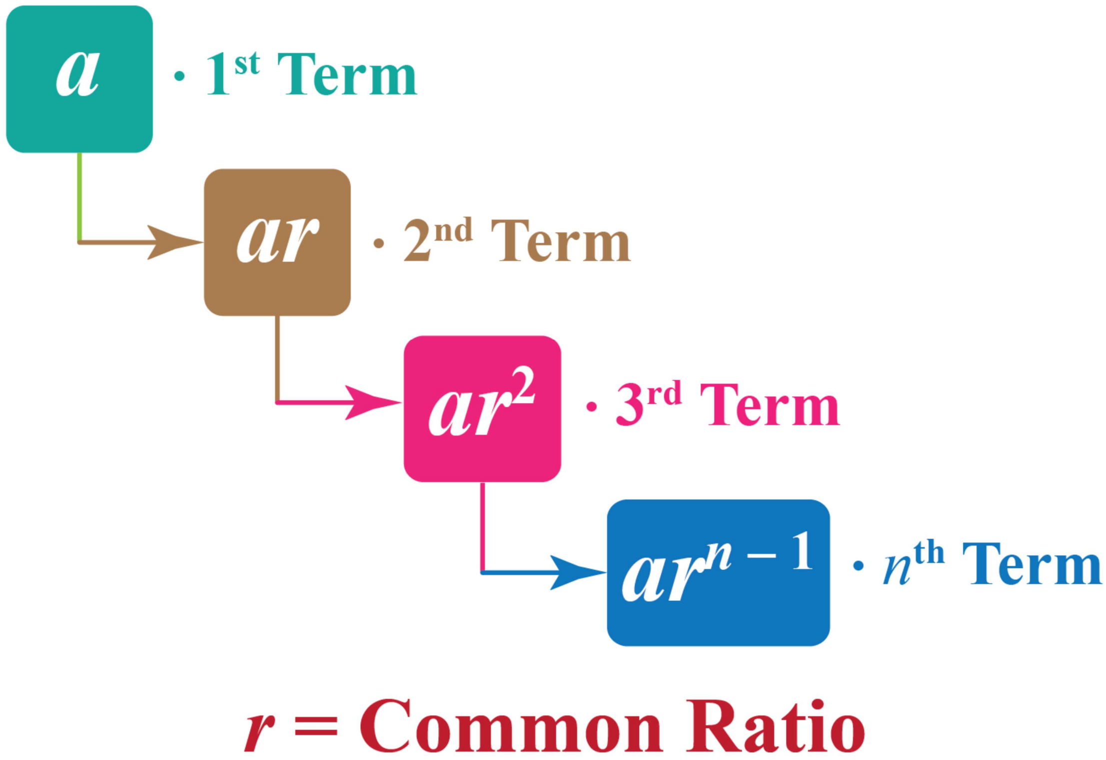



*Figure 2.7. Illustration of common ratio (Source: Kofa study, Creative Commons License)*

#### 2.4.1.2. The Common Ratio's Impact on the Sequence

The magnitude and sign of the common ratio ($r$) significantly influence the nature of the geometric sequence:

1. **Exponential Growth ($\boldsymbol{r > 1}$):** When the common ratio ($r$) is greater than 1, the sequence exhibits exponential growth. Each term is larger than the previous one by a factor of ($r$). This behavior is characteristic of phenomena involving rapid expansion, such as population growth or compound interest.
2. **Exponential Decay ($\boldsymbol{0 < r < 1}$):** Conversely, when the common ratio ($r$) falls between 0 and 1, the sequence undergoes exponential decay. Each term becomes smaller than the preceding one by a factor of ($r$). This pattern is observed in processes like radioactive decay or diminishing returns in investments.

3. **No Growth or Decay ($r = 1$):** In the special case where the common ratio ($r$) equals 1, the sequence remains constant. In this scenario, each term is equal to the preceding term, signifying no growth or decay.

**Example: Exponential Growth**

Consider a scenario where you invest $1,000 in a savings account that provides an annual interest rate of 10 %. Using the formula for a geometric sequence, we can model the growth of your investment over several years (Johansen & Sornette, 2001).

In this example, ($a_1$) represents the initial investment of $1,000, and the common ratio ($r$) is 1 plus the annual interest rate (expressed as a decimal), which is ($0.10 + 1 = 1.10$). We want to find the value of the investment after different years.

After 1 year ($n = 2$), the investment becomes:

$$a_2 = 1{,}000 \cdot 1.10^{(2-1)} = 1{,}100 \text{ dollars.}$$

After 2 years ($n = 3$), it grows to:

$$a_3 = 1{,}000 \cdot 1.10^{(3-1)} = 1{,}210 \text{ dollars.}$$

After 3 years ($n = 4$), it further increases to:

$$a_4 = 1{,}000 \cdot 1.10^{(4-1)} = 1{,}331 \text{ dollars.}$$

This sequence demonstrates the concept of exponential growth, where each term is calculated by multiplying the previous term by the common ratio of 1.10.

**Example: Exponential Decay**

Suppose you have a radioactive substance with an initial mass of 1 gram, and it decays at a constant rate of 20% per hour. We can model the decay of this substance over time (Arroyo et al., 2011).

In this scenario, ($a_1$) represents the initial mass of 1 gram, and the common ratio ($r$) is 1 minus the decay rate (expressed as a decimal), which is ($0.20$). We aim to find the remaining mass after different hours.

After 1 hour ($n = 2$), the mass becomes:

$$a_2 = 1 \cdot (1 - 0.20)^{(2-1)} = 0.80 \text{ grams.}$$

After 2 hours ($n = 3$), it further decreases to:

$$a_3 = 1 \cdot (1 - 0.20)^{(3-1)} = 0.64 \text{ grams.}$$

After 3 hours ($n = 4$), it continues to decay:

$$a_4 = 1 \cdot (1 - 0.20)^{(4-1)} = 0.512 \text{ grams.}$$

This sequence illustrates the concept of exponential decay, where each term is calculated by multiplying the previous term by the common ratio of 0.80.

The common ratio is a fundamental parameter that underpins the behavior of these sequences, enabling us to model and understand phenomena involving consistent change over time. From finance to biology, physics to engineering, and beyond, the concept of the common ratio in geometric sequences stands as a powerful tool for describing the dynamics of exponential processes in the natural and mathematical world (Barndorff-Nielsen & Shephard, 2001).

### 2.4.1.3. Determining the 10$^{th}$ Term

To further illustrate the concept of the common ratio within geometric sequences, let's explore a specific example to determine the 10$^{th}$ term of a geometric sequence. This example will highlight how the common ratio governs the progression of the sequence (Gura, 1996).

**Scenario:** Consider a geometric sequence where the first term $(a_1)$ is 3, and the common ratio $(r)$ is 2. We want to find the 10$^{th}$ term $(a_{10})$ of this sequence.

Using the formula for a geometric sequence:

$$a_n = a_1 \cdot r^{(n-1)}$$

Here, we have:

$a_1 = 3$ (the first term)

$r = 2$ (the common ratio)

$n = 10$ (we want to find the 10th term)

Now, let's calculate $(a_{10})$ using the formula:

$$a_{10} = 3 \cdot 2^{(10-1)}$$

Simplify the exponent:

$$a_{10} = 3 \cdot 2^9$$

Calculate $(2^9)$, which is 512:

$$a_{10} = 3 \cdot 512$$

Finally, multiply 3 by 512 to find the 10th term:

$$a_{10} = 1{,}536$$

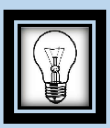
So, the 10th term ($a_{10}$) of this geometric sequence with a first term of 3 and a common ratio of 2 is equal to 1,536.

In this example, the common ratio ($r = 2$) played a crucial role in determining the 10th term. It represented the factor by which each term was obtained from the previous one. As we can see, the progression was not linear; each term was twice the value of the preceding term, leading to exponential growth. This is a characteristic of geometric sequences when the common ratio is greater than 1 (Qonita & Wuryandari, 2015).

The 10th term, 1,536, is a testament to the power of exponential growth. Starting with the first term of 3, each subsequent term grew significantly larger due to the common ratio of 2. This property is why geometric sequences are valuable for modeling processes involving exponential change, such as investments with compound interest or population growth (Qonita et al., 2015).

## 2.4.2. Calculating the $n^{th}$ Term of a Geometric Sequence

This capability empowers us to determine the value of any term in the sequence without having to compute all the preceding terms individually. In this exploration, we will discuss the methodology for calculating the $n^{th}$ term of a geometric sequence, presenting the mathematical framework behind it and providing illustrative examples.



**Tip:** When calculating the nth term of a geometric sequence $\left(a\ sub\ n = a\ sub\ 1 * r^{(n-1)}\right)$, pay close attention to the common ratio (r) and the initial term (a sub 1). Small changes in these values can lead to significant differences in the sequence's behavior.

### 2.4.2.1. The General Formula

To calculate the $n^{th}$ term of a geometric sequence, we utilize the general formula provided above. This formula encapsulates the essence of geometric sequences and allows us to compute any term within the sequence with ease (Angraini et al., 2023).

**Calculating the $n^{th}$ Term:**

Consider the following geometric sequence: 2, 6, 18, 54, 162, ...

In this sequence:

i. The first term ($a_1$) is 2.

ii. The common ratio $(r)$ is found by dividing any term by the preceding term. For instance, $(\frac{6}{2} = 3)$, $(\frac{18}{6} = 3)$, and so on. Therefore, $(r = 3)$.

Now, suppose we want to find the 7th term $(a_7)$ of this sequence. We can use the general formula:

$$a_n = a_1 \cdot r^{(n-1)}$$

In this case:

$$a_1 = 2$$

$$r = 3$$

$$n = 7$$

Now, let's calculate $(a_7)$:

$$a_7 = 2 \cdot 3^{(7-1)}$$

Simplify the exponent:

$$a_7 = 2 \cdot 3^6$$

Calculate $(3^6)$, which is 729:

$$a_7 = 2 \cdot 729$$

Finally, multiply 2 by 729 to find the 7th term:

$$a_7 = 1458$$

So, the 7th term $(a_7)$ of this geometric sequence is 1458.

#### 2.4.2.2. Understanding the Formula

The formula for calculating the n^th^ term of a geometric sequence is derived from the inherent multiplicative pattern within these sequences. Each term is a result of multiplying the preceding term by the common ratio $(r)$. Therefore, to find any term in the sequence, we start with the first term $(a_1)$, then apply the common ratio $(r)$ repeatedly for $(n-1)$ times, as the term number dictates (Tong et al., 2021).

Calculating the $n^{th}$ term of a geometric sequence is fundamental and enables one to understand and model exponential growth or decay patterns in various domains. The formula, derived from the inherent multiplicative nature of these sequences, simplifies the process of determining specific terms within the sequence, making it a valuable tool for solving practical problems and making informed decisions. Geometric sequences, with their predictable progression, continue to be a cornerstone in mathematics